Background and Theory

To solve complex differential equations, to which no mathematical solution is known, one has a few options. Today, approximations done digitally are the industry standard. An interesting alternative is to build an electronic circuit which solves the differential equation by analogizing the dependant variable with respect to time. This competition will compare these two methods.

The circuit for this competition will be a an electrical simulation of a spring-mass system modeled by the differential equation $x'' + \frac{\beta}{M} \bullet x' + \frac{k}{M} \bullet x = 0$ or $x'' = -\frac{\beta}{M} \bullet x' - \frac{k}{M} \bullet x$. M is the mass of the object, k is the spring constant, and β is the damping coefficient. Figure 1 shows the block diagram for the solution of the differential equation.

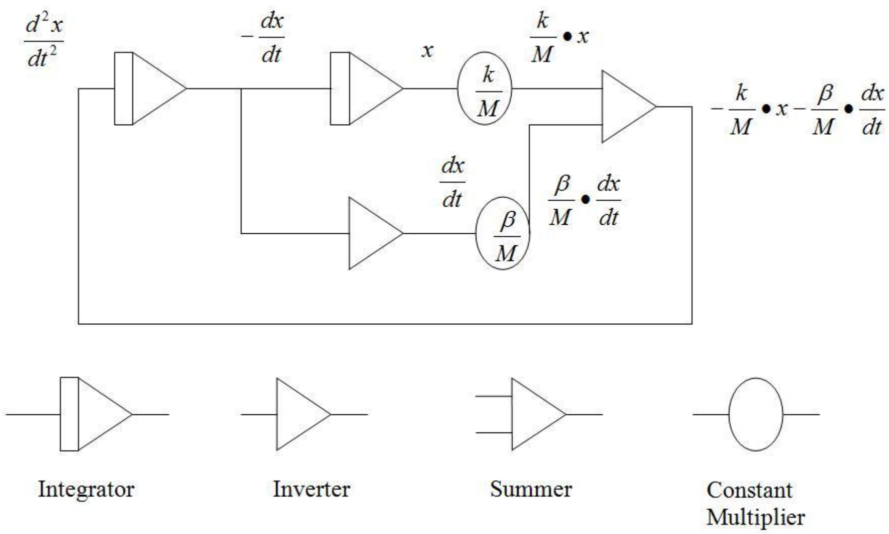


Figure 1

Conveniently, the operational amplifier can be utilized for every operation shown in the Figure 1. Figure 2 shows the circuit to realize the solution.

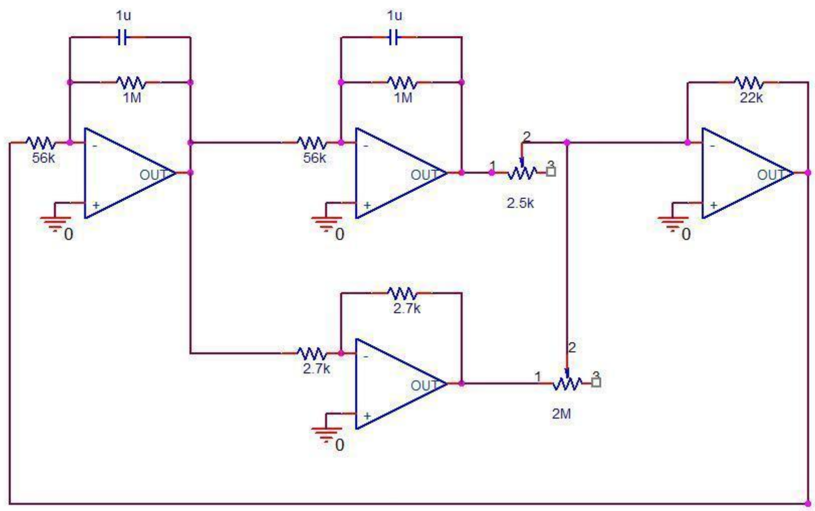


Figure 2

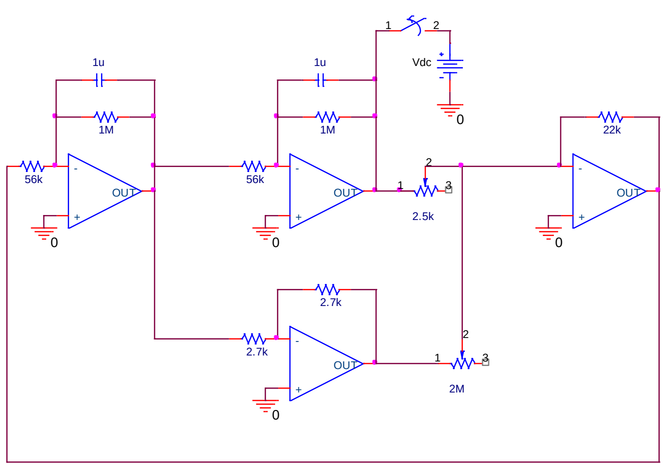
Electrically, the voltage at each point represents the value, shown in the block diagram, at that point. The circuit's resting state, just like a spring-mass, is zero for x, x', and x''. **In order to do any simulation, initial conditions must be set at the appropriate points.**

Objectives

The following can be checked off in any order.

1. Build the circuit shown in Figure 2 and demonstrate its operation.

As stated in the description, the points x, x', and x'' set at zero voltage when no initial condition is set. One method of setting the initial condition is to use a switch or a piece of wire to temporarily connect point x to a DC voltage. Releasing the voltage causes a damped sinusoid at point x. Applying and releasing a DC voltage is analogous to stretching and releasing a spring mass system.



2. The circuit in Figure 2 operates at faster than real time. **Modify the circuit** to increase or decrease the speed.

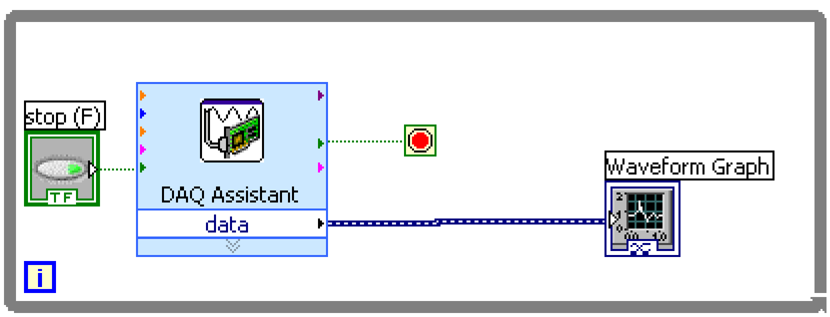
The rate of integration of the two op amp integrators is determined by:

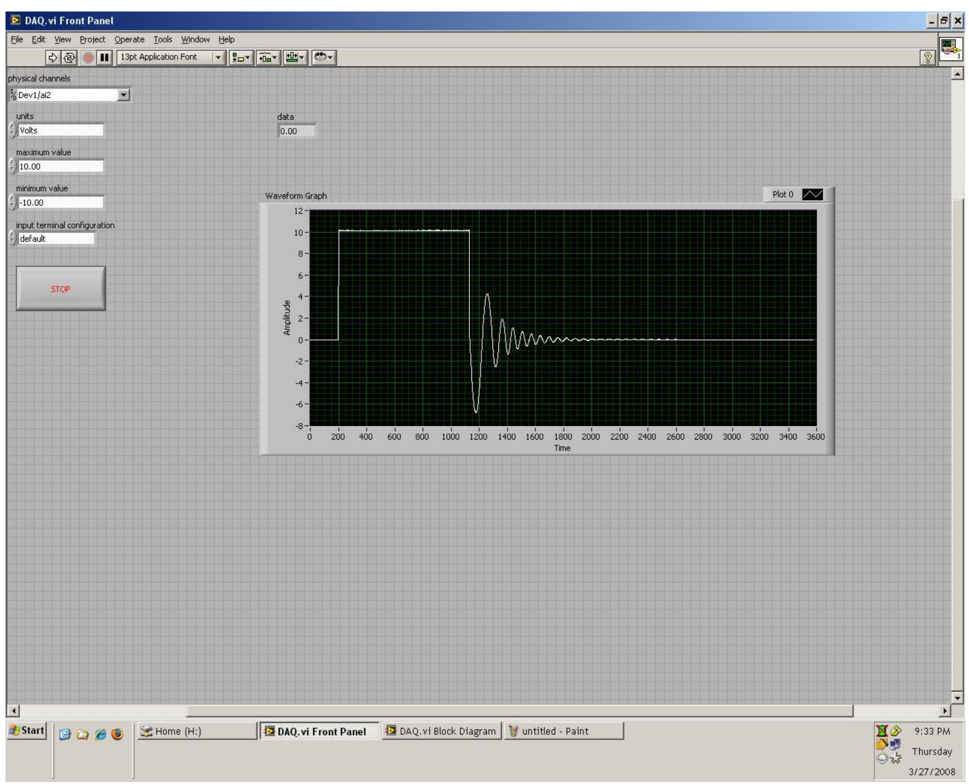
$$\frac{1}{RC} = \frac{1}{56k\Omega * 1\mu F} = 17.857$$

Changing either the 1uF cap or 56k resistor will change the rate of integration.

3. Use LabVIEW and the NIDAQ hardware provided to sample x and display the waveform on the front panel.

The LabVIEW portion can be completed several ways, one of which is as follows:





4. Use SciLab to find a numerical solution to the values given.

$x(0) = 2$

$x'(0) = -1$

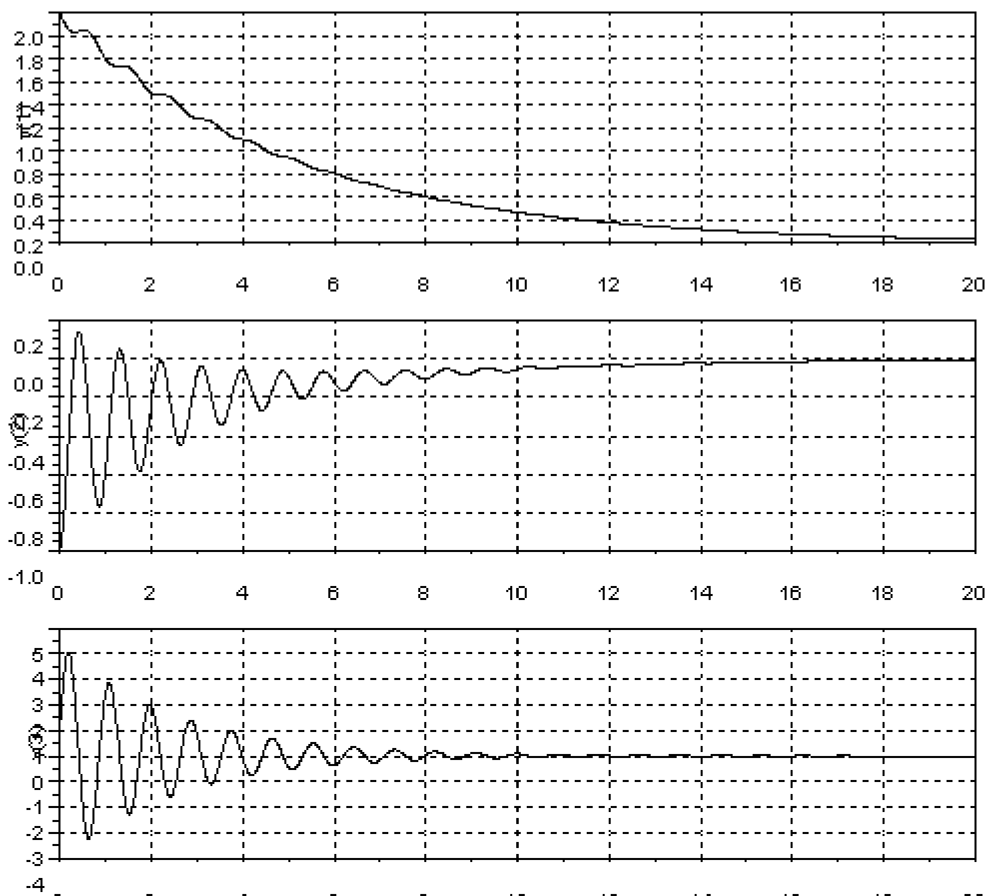
$x''(0) = 0$

$M = 1$

$\beta = 50$

$k = 10$

One way to go about solving this problem is to assume that if y''' is known than we can integrate it three times to get our solution. We must remember to solve for y''' and keep all the constants of the original ordinary differential equation. All initial conditions must be set and the "ode" function in SciLab can be utilized to solve the differential equation.



```
function [yprim] = diffeq(t,y)
   yprim(1) = y(2);
   yprim(2) = y(3);
   yprim(3) = -y(3) - 50 * y(2) - 10 * y(1);
endfunction



// Set the initial conditions for the DE here
y(1) = 2; y(2) = -1; y(3) =0;
y0 = [y(1); y(2); y(3)];

t0 = 0;
t = 0:.01:20;



// Solve the ODE and send the result to y.
y = ode( y0, t0, t, diffeq);

xset('window',1);xbasc();
subplot(3,1,1)
plot2d(t,y(1,:));xgrid()
xtitle( "","time","y(1)");
subplot(3,1,2)
plot2d(t,y(2,:));xgrid()
xtitle( "","time","y(2)");
subplot(3,1,3)
plot2d(t,y(3,:));xgrid()
xtitle( "","time","y(3) ");
```